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Methodological Shortcomings in Classical Mathematical Economics: A Critical Examination of the Contributions of Cournot, Jevons and Walras

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Abstract

This paper examines critically the contributions of Cournot, Jevons and Walras as the founders of classical mathematical economics from a methodological standpoint. Advances in different economic schools and doctrines in the 19th century produced an environment of multi-dimensionality in economic analysis which was regarded by the pioneers of classical mathematical economists as a chaotic state. We have demonstrated that the formation of this new discipline, known equivalently as pure or scientific economics, was a response to this so-called chaotic state. We have also shown that the erroneous logic of abstraction in the sense of reducing a multi-dimensional economic system to a one-dimensional mechanical framework as the methodological basis of classical mathematical economics has been the origin of serious shortcomings in mathematical treatment of economics. Based on the writings of Jevons and Walras we have provided evidences to support the claim that advances in Marxian economics can be considered as the prime motive in the development of classical mathematical economics.

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1. Introduction

While physical sciences can hardly develop without mathematics, the application of mathematical methods in economic analysis has remained a controversial issue. Let us initiate the argument by proposing the following questions: Has any economic fact of significant value been discovered by the application of mathematical methods? Is economics a branch of science because it is mathematical in nature or has the mathematical treatment of economic issues ranked economics as a branch of established sciences? What is the logical justification for the application of mathematical methods to economic analysis from a methodological stand point? What are the salient features of mathematical methods which have made them so attractive to the community of mathematical economists and econometricians? Why eminent classical economists like Smith, Say, Ricardo and Mill did never apply mathematical methods in their work? Why some modern economists of great reputation and with strong mathematical background, like John Maynard Keynes, were not interested in mathematical economics? What factors have contributed to the partial successful applications of mathematical methods to economic analysis and what have been the underlying causes for its partial failure?

The aim of this paper is not to provide answers to the above-mentioned questions. In fact, these and many more questions related to mathematical treatment of economics cannot be successfully attended without direct reference to the origin and methodological shortcomings of mathematical economics.

We have classified the literature on mathematical economics into three broad categories as follows: *i*) The early mathematical treatment of economic problems which includes 38 published work starting from the contribution of Civa (1711) on money[1] to the path-breaking work of Cournot (1838)[2]. *ii*) The classical mathematical economics originating from the contributions of Jevons (1871) and Walras (1874). I have included Cournot (1838) in this category as well due to its profound theoretical significance as well as its impact on the later development in mathematical economics. It should be mentioned however, that what is known today as the neoclassical economics is an extended literature originating from the classical economics, which also covers a wide-range of topics in modern mathematical economics. *iii*) The modern mathematical economics which covers the literature on mathematical

treatment of economics from the 1930's to the present time, whose analysis is beyond the scope of this paper.

An examination of the available explanations regarding the origin of classical mathematical economics is presented in Derakhshan (2014), where the literature on this topic are critically examined with reference to four categories of arguments put forward by Debreu (1986, 1987), Cournot (1838), Walras (1874) and von Neumann and Morgenstern (1944). Hence, this paper deals mainly with methodological considerations from a historical standpoint regarding the contributions of Cournot, Jevons and Walras as the pioneers in classical mathematical economics. However, within this methodological context I have also proposed my own explanations of the origin of classical mathematical economics as will be discussed in Section 5.

The underlying factors of classical mathematical approach to economic analysis are the subject matter of Section 2 where it is shown that the pioneers of classical mathematical economics defined this approach as scientific approach in order to demonstrate that it's potential in advancing economic analysis is similar to the advancement in physical sciences resulting from the applications of mathematical methods. From a methodological point of view, classical mathematical economics and the formation of mechanical economic science are discussed in Section 3.

A critical analysis of the sub-divisionism approach in classical mathematical economics as a remedy to multi-dimensional political economy is presented in Section 4. Historically, the formation of different schools of thoughts and doctrines in the 19^{th} century and particularly the diffusion of socialism produced an environment of multi-dimensionality approach in economic analysis which was regarded by the founders of classical mathematical economics as a *chaotic state*. We have demonstrated in this section that the so-called chaotic state in economics had a profound impact on the formation of "scientific or mathematical" approach to economic analysis. It is also shown that the erroneous reduction of multi-dimensional real-life economic performance to an abstract simple mechanical economic behavior of representative individuals amenable for mathematical manipulation has been the origin of serious methodological shortcomings in classical mathematical treatment of economics.

In Section 5, we have proposed the idea that the rapid theoretical development in socialism in general and Marxian economics in particular

in the 19th century had a prime significant impact on the formation of classical mathematical economics. Summary and concluding remarks are the subject matter of Section 6.

2. Underlying Factors of the Scientific or Classical Mathematical Approach to Economic Analysis

The growing desire for reducing the multi-dimensional economic studies to a one-dimensional mechanical framework in the early nineteenth century strongly motivated the mathematical treatment of economics. Let us examine briefly the underlying factors for this motivation.

1. Advances in classical economics in the first half the 19th century, and particularly the contributions of Smith (1776), Ricardo (1817) and Mill (1848), together with the developments in theoretical socialism in general and Marxian economics in particular [Marx (1848, 1859 and 1867)][3], produced an environment of multi-dimensionality in economic analysis in which economic issues were studied in relation to historical, social, cultural and political observations. This environment, which was labeled as *chaotic state* by the early advocates of mathematical economics, induced a "scientific, mathematical or pure" approach to economic analysis.

According to Cournot (1838, the first paragraph in the preface), "The science known as Political Economy, which for a century has so much interested thinkers, is to-day more generally diffused than ever before … [and attracted] the attention of the great journals, which are to-day the most important means of spreading information; but the public is so tired of theories and systems that now the demand is for so-called "positive" matters … such as will throw the light of experience on the important questions which are being agitated before the country and which so greatly interest all classes of society."[4]

Jevons in a lecture on *The Future of Political Economy,* delivered at the University College, London in 1876 maintained that "One hundred years after the first publication of the *Wealth of Nation,* we find the state of the science to be almost chaotic. There is certainly less agreement now about what political economy is than there was thirty or fifty years ago." Similar idea is expressed in Jevons (1879)[5]: "The present chaotic state of Economics arises from the confusing together of several branches of knowledge."

2. Despite the fact that the inherent inter-relations in different aspects of social life make separate studies of any of them not sufficiently productive, the rapid growth in physical sciences in the 19th century had a profound impact on many authors to search for an economic science similar to physical sciences. Moreover, the advocates of mathematical approach to economics recommended strongly that economists should assume their distinctive role of analyzing pure economic issues and avoiding the study of the laws of a unified social science.

According to Jevons (1871, p. 20), "… instead of converting our present science of economics into an historical science, utterly destroying it in the process, I would perfect and develop what we already possess and at the same time erect a new branch of social science on an historical foundation. This new branch of science ... is doubtless a portion of what Herbert Spencer calls Sociology, the Science of the Evolution of Social Relations."

3. It was generally agreed that the rapid progress in physical sciences in the 19th century was mainly due to the breaking-up of broad problems into their component parts. This outlook motivated the advocates of mathematical approach to economics to accept the view that political economy should no longer be seen as if it were a single undividable discipline.

4. The first step towards making a science of economics in the same fashion as physical sciences was believed to be the discovery of general laws of economics which remain the same throughout all different ages and conditions: "Just as there is a general science of mechanics, so we must have a general science or theory of economy. ... The theory of economy proves to be, in fact, the mechanics of utility and self-interest" [Jevons (1876)][6]. It is not surprising therefore that economic science became identified with mathematical or *pure* economics. Historically, advances in mechanics had the greatest impact on the formation of classical mathematical economics, hence the role of calculus in mathematical treatment of economics has always been profound.

5. Abstracting a pure economic sub-system from the real-life performance as distinct from philosophical, historical, political and social sub-systems was regarded by classical mathematical economists as a remedy to the prevailing state of multi-dimensionality (or the so-called chaotic state) in economic analysis. The concept of *pure* or *mathematical economics*, which was identical to *economic science*, was then naturally

emerged to represent a positive or ideologically neutral system of economic knowledge.

3. Classical Mathematical Economics and the Formation of Mechanical Economic Science

Classical mathematical economics, in the sense discussed above, was developed in conjunction with the concept of *pure economics.* In fact, mathematical economics in general cannot be defined properly without using the concept of pure economics and vice versa. This will play an important role in the understanding of underlying factors in the limitations of mathematical approach to economics. In this section, we first examine how the instrumentality of mathematics has played the critical role in the formation of mechanical economic science before considering, in Section 4, that the prime objective of the pioneers of classical mathematical economics was to demonstrate that the mathematical treatment of economics was in fact a remedy to the multi-dimensional political economy.

According to Jevons (1871, p. *vii*, preface), since economics "deals throughout with quantities, it must be a mathematical science in matter if not in language. ... The Theory of Economy thus treated presents a close analogy to the science of Statistical Mechanics and Laws of Exchange are found to resemble the Laws of Equilibrium of a lever … The nature of Wealth and Value is explained by the consideration of indefinitely small amounts of pleasure and pain, just as the theory of Statics is made to rest upon the equality of indefinitely small amounts of energy." Furthermore, on page 3 (*ibid*) he writes that "It is clear that economics, if it is to be a science at all must be a mathematical science ... My theory of Economics, however, is purely mathematical in character. "

By mathematics, Jevons basically meant differential calculus: "The theory consists in applying the differential calculus to the familiar notions of wealth, utility, value, demand, supply, capital, interest, labor and all the other quantitative notions belonging to the daily operations of industry." (*ibid*, p. 3). However, he held that the minimization of costs in fulfilling the utility of an individual is the ultimate objective of economic science which he defined as the *Calculus of Pleasure and Pain* (1871, p. *vi*, preface). He wrote on page 27 (*ibid*) that "the calculus of utility aims at studying the ordinary wants of man at the least cost of labor."

We should briefly mention here that although Jevons has been usually praised for introducing into economic analysis the idea of maximization (or minimization), the origin of these ideas traces back to Cournot in 1838 when he wrote on page 44 in chapter 4 of his book that "we shall invoke but a single axiom, or, if you prefer, make but a single hypothesis, i.e. that each one seeks to derive the greatest possible value from his good or his labor."

The modern definition of economics as the allocation of scarce resources for optimum satisfaction of alternative needs is in fact the generalization of Jevons's calculus of pleasure and pain. Moreover, the modern mathematical optimization techniques, which are the most efficient mathematical tools for achieving optimum satisfaction of needs, are nothing but the advanced versions of the elementary calculus employed by Jevons.

The nature of general mathematical methods in economic analysis has been remained basically unchanged since Jevons although its role and scope has been remarkably extended. According to Jevons (1879: 2nd edition of 1871), "... the method consists in assuming certain simple conditions of the functions as conformable to experience and then disclosing by symbolic inference the implicit results of these conditions." (p. *xxxi*, preface). As for the role of mathematics, we refer to Fisher (1891, p. 119) where he states that "The effort of the economist is to *see*, to picture the interplay of economic elements. ... Mathematics is the lantern by which what before was dimly visible now looms up in firm, bold outlines. We see better. We also see further."

It is interesting to note that this idea can also be traced back to Cournot (1838). According to him the objective of using mathematical symbols is "to facilitate the exposition of problem, to render it more concise, to open the way to more extended developments and to avoid the digressions of vague argumentation." (p. 3). A simple comparison of Cournot's definition of the objective of mathematical economics with similar modern definitions reveals the fact that there has not been any significant change since then. For example, the Editors of the *Journal of Mathematical Economics*, (2014), express the *Journal's* statement of aims as follows: "In the Editors view, the formal mathematical expression of economic ideas is of vital importance to economics. Such an expression can determine whether a loose economic intuition has a

coherent, logical meaning. Also, a full formal development of economic ideas can itself suggest new economic concepts and intuitions."

4. Critical Analysis of the Logic of Abstraction in Classical Mathematical Economics: The Role of Sub-divisionism in Reducing Multi-dimensional Economic Analysis to a One-dimensional Mechanical Framework

We discussed in Section 2 that from the early 19th century the advocates of mathematical treatment of economics regarded the prevailing philosophical and historical approach to economic analysis as a chaotic state in political economy. Sub-divisionism approach, i.e. the reduction of multi-dimensional economic analysis to a one-dimensional mechanical framework of classical mathematical economics was considered to be a remedy to this chaotic state. According to Walras (1874, English translation 1954, p. 471), "There are today heaven knows how many schools of political economy: the *deductive* school and the *historical* school, the school of *laisser-faire* and the school of *State intervention* or *Socialism of the Chair,* the *Socialist school* properly so-called, the *Catholic* school, the *Protestant* school, etc. For my part, I recognize only two: the school of those who do not demonstrate and the school, which I hope to see founded, of those who do demonstrate their conclusions. By demonstrating rigorously first the elementary theorems of geometry and algebra and then the resulting theorems of the calculus and mechanics, in order to apply them to experimental data, we have achieved the marvels of modern industry. Let us follow the same procedure in economics and, without doubt, we shall eventually succeed in having the same control over the nature of things in the economic and social order as we already have in the physical and industrial order."

For Cournot, this sub-divisionism meant constructing a pure or positive economics which should be developed independent of the prevailing political systems: "I will only observe that *theory* ought not to be confounded with systems ... and that, to a man of my position in particular, more than to any other, it should be permissible to consider from an exclusively theoretical standpoint, a subject of general interest which has so many different sides." (Cournot, 1838, pp. 1-2, preface).

Jevons (1871, p. 20) clearly signifies the necessity for subdividing economic knowledge: "Political economy is in a chaotic state at present, because there is need for subdividing a too extensive sphere of

knowledge." According to Jevons, "we must distinguish the empirical elements from the abstract theory, from the applied theory, and from the more detailed art of finance and administration. Thus will arise various sciences, such as commercial statistics, the mathematical theory of economics, systematic and descriptive economics, economic sociology and fiscal sciences … Then will be division according to the manner of treating the branches of subject. The manner may be theoretical, empirical, historical, or practical; the subject may be capital and labor, currency, banking, taxation, land tenure, etc. -and not to speak of the more fundamental division of the science as it treats of consumption, production, exchange and distribution of wealth." (*ibid*, 2nd edition, 1879, p. *xvii*).

Differentiating between scientific and literary (or non-mathematical) temper in economic analysis can best be seen in Walras (1874) who named his book *Elements d' Economie Politique Pure.* The fact that he was indebted to Cournot (1838) for using calculus in economic analysis implies that by pure economics he basically meant mathematical economics. On page 37 in the preface to the 4th edition of his book, Walras maintains that "I readily acknowledge Gossen's priority[7] with respect to the utility curve and Jevons's priority with respect to the equation of maximum utility in exchange, but these economists were not the source of my ideas. I am indebted to my father, Auguste Walras, for the fundamental principle of my economic doctrine and to Augustin Cournot for the idea of using the calculus of functions in elaboration of this doctrine."Walras interchangeably used the terms mathematical economics and scientific economics to explain pure economics. This can clearly be seen in the introduction to the English translation of Walras's book *Elements of Pure Economics* in which William Jaffe, the translator, wrote in 1954 about how the economist Auguste Walras asked his son, Leon, to study mathematical economics at the age of 24 in order to build up a *scientific economics.*

4.1 The Erroneous Logic of Abstraction

Walras can be considered as the first mathematical economist who clearly defined and examined the problem of *abstraction* in pure (or mathematical) economics. Being impressed by the advances in physical sciences, he argued (1874, p. 71) that "From real-type concepts, these sciences abstract ideal-type concepts which they define and then on the

basis of these definitions they construct *a priori* the whole framework of their theorems and proofs. After that they go back to experience not to confirm but apply their conclusions." The surprising fact that Walras was not looking for any confirmation of the proposed mathematical model stems from his perception that "reality confirms these definitions and demonstrations only approximately and yet reality admits of a very wide and fruitful applications of these propositions." (*ibid,* p. 71) Following the same procedure, Walras defined the pure theory of economics as a science which "ought to take over from experience certain type concepts, like those of exchange, supply, demand, market, capital, income, productive service and products. From these real-type concepts the pure science of economics should then abstract and define ideal-type concepts in terms of which it carries on its reasoning." However, Walras has taken three different positions regarding the aim of pure economics, which are at variance with his method of abstraction presented above.

1- Walras maintains that it is not the aim of pure economics to provide solutions to real-life economic problems; it only furnishes an academic pleasure to an economic scholar: "To be sure, a scholar has a right to pursue science for its own sake, just as the geometer has the right (which, in fact, he exercises every day) to study the most singular properties of geometrical figures, however fantastic, if he finds that they excite his curiosity. " (*ibid*, p. 71-72)

2- At the same time he admits that pure economics cannot only solve real economic problems but can control the nature of things in economics and social order exactly in the same manner as physical and industrial order are in control: "By demonstrating rigorously first the elementary theorems of geometry and algebra and then the resulting theorems of the calculus and mechanics, in order to apply them to experimental data, we have achieved the marvels of modern industry. Let us follow the same procedure in economics and, without doubt, we shall eventually succeed in having the same control over the nature of things in economics and social order as we already have in the physical and industrial order." (*ibid*, p. 471)

3- Walras has also taken a conservative position. He has reduced the difficulties associated with applications of mathematical methods simply to a set of technical complications which can easily be treated by other

economists in due course: "... practically all the criticisms leveled against me have consisted in calling my attention to complications which I had left to one side. I find it very easy to reply to these criticisms. So far as I am concerned, since I was the first to elaborate a pure theory of economics in mathematical form, my aim has been to describe and explain the mechanism[s] ... in terms of [their] bare essentials. It is for other economists who come after me to introduce one at a time whatever complications they please. They in their way and I in mine will then, I think, have done what had to be done." (*ibid,* p. 478)

Unfortunately, Walras has ignored the very important issue of the method of abstracting the "ideal-type" economic concepts from the real-type concepts which embrace the increasing number of complexities existing in real-economic life and at the same time maintaining its abstract nature, which is so essential for mathematical treatment of economic behavior[8]. This is an important problem to which I shall refer briefly as follows.

We mentioned in the beginning of this paper that the application of mathematical methods in economics is not yet a settled problem. The question arises as to what extent has this been due to the fact that mathematicians were not well acquainted with economics or economists were not good mathematicians? Our analysis in the previous sections implies that none of these can provide a satisfactory answer. It seems that the logic of abstraction in the methodology of mathematical economics plays the key role.

Let us confine the argument to very simple heuristic assertions. It is easy to see that mathematics is a system of logical reasoning based on *abstract* notions. No single topic in economics can be treated mathematically without first being reduced to abstract and narrow concepts and then being fed into the mathematical machinery in order to infer necessary logical conclusions. Since economic input to mathematical machinery is abstract, the output will also become abstract.

Under what conditions can one obtain economic results of value using mathematical reasoning? To answer this question, I first assume that the value of a result depends on its explanatory power, either being useful in explaining some other unknown theoretical economic facts or being able to explain a real world economic observation. The former is a contribution to pure or mathematical economics and the latter constitutes contributions to applied economics.

An examination of the classical works on mathematical economics reveals the fact that their ultimate goal was primarily to discover the dynamics of "pure economics" without direct reference to the real world economic issues. This would certainly permit an endless fascinating theoretical journey in mathematical economics. An inspection of papers published in specialized journals in the field of mathematical economics supports this argument.

However, a point of theoretical significance as well as practical importance in economic theorization is how to identify the conditions under which mathematical treatment of economics can lead to results of value in explaining the real world economic issues. The methodology employed in deriving abstract notions from the real world economic life plays the key role. A real economic problem cannot properly be studied in isolation of the related historical, political, sociological and cultural contents. The attempts by Cournot, Jevons and Walras in establishing pure or mathematical economics in isolation of other related dimensions were the first erroneous move in the process of abstraction in economic theorization. This together with the simplifying assumptions which are usually made to facilitate the applications of more advanced mathematical methods have produced the existing rich literature in mathematical economics and yet unproductive in addressing the real world economic problems.

The erroneous method of abstraction employed by classical mathematical economists was nothing more than a simple division of multi-dimensional political economy into different disciplines: "Political economy is in a chaotic state at present, because there is need for subdividing a too extensive sphere of knowledge." [Jevons (1871), p. 20]. It seems unlikely that the analytical results obtained from the behavior of an abstract fragmented part of a multi-dimensional political economy can truly represent the behavior of the system as a whole. The results obtained in one-dimensional mathematical economics are valid only within its own domain and cannot by itself provide results of value for the real multi-dimensional economic problem. A proper method of abstraction should therefore reduce complexities existing in the real-life economic performance while preserving the underlying properties of relations with other sub-systems.

It follows therefore that applications of mathematical methods to economic analysis are most promising in those areas where the abstract

economic notions to be used in mathematical machinery constitute close approximations to economic realities. These instances, which are likely to be found in technical issues in microeconomics and finance, are least affected by the underlying non-economic factors such as political, cultural, and historical elements. This explains why these instances are always referred to as the successful examples in the applications of mathematics to economics.

5. The Impact of Marxian Economics on the Formation of Classical Mathematical Economics

We discussed in Section 1 that the formation of different schools of thoughts and doctrines in the 19th century and particularly the diffusion of socialism produced an environment of multi-dimensionality approach in economic analysis which was regarded by the founders of classical mathematical economics as a *chaotic state*. We argued further that the so-called chaotic state in economics had a profound impact on the formation of "scientific or mathematical" approach to economic analysis. Based upon the writings of Jevons and Walras, we now claim that the diffusion of Marxian economics established the strongest motive for these writers to create the new discipline of classical mathematical economics.[9] Let us first refer to Marx's main contributions published before Jevons (1871) and Walras (1874), which include *Manifesto of the Communist Party* (1848), *A Contribution to the Critique of Political Economy* (1859) and *Capital, A Critique of Political Economy*, vol. 1 (1867).[10]

The point of prime importance is that Marxian economics is an integrated body of knowledge which aims at studying real-life economic issues in connection with the philosophical dimension, i.e. the dialectical materialism and the historical dimension, i.e. the materialist concept of history. In Marxian tradition every fundamental economic concept, such as value or capital, can best be explained within this multi-dimensional space. By concentrating only on the abstract economic dimension and ignoring the feedback mechanism with philosophical and historical aspects, pure or mathematical economics depleted the real contents of fundamental economic concepts. This naturally reduced the aim of economic analysis to a simple logical or mathematical inference based on a number of basic simplifying assumptions which were laid upon the empty concepts.

An examination of the concept of value may highlight the above-mentioned point. The concept of value in Marxian economics with its class dimension was reduced to a superficial concept related to price, pleasure or utility in the classical mathematical economics. In this approach, class had been replaced by an individual economic agent, thus the historical dimension associated with class formation (the historical materialism) and its relation to value (the formation of surplus value and exploitation) became effectively futile or irrelevant within the context of pure or mathematical economics.

To provide evidences for the above analysis, let us refer to Jevons (1871, p. 1) who defined utility as the origin of value: "Repeated reflections on inquiry have led me to the somewhat novel opinion that *value depends entirely upon utility.* Prevailing opinions make labor rather than utility the origin of value; and there are even those who distinctly assert that labor is the *cause* of value. Labor is found often to determine value, but only in an indirect manner, by varying the degree of utility of the commodity through an increase or limitation of the supply."

On the basis of a one-dimensional concept of utility, Jevons (1871, p. 1) defined his aim as to find the laws of variations of utility and to derive a theory of exchange on the basis of utility: "We have only to trace out carefully the natural laws of the variations of utility, as depending upon the quantity of commodity in our possession, in order to arrive at a satisfactory theory of exchange, of which the ordinary laws of supply and demand are a necessary consequence." Reducing the analysis of value to a one-dimensional framework was further advanced when Jevons in his *Principle of Economics,* which was published after his death in 1905, stated that "... as value, after all, is but a development of utility, I have seen reason to take utility rather than value as the subject-matter of economics." (p. 49)

The implications of this approach for historical, political, philosophical and social aspects of economic analysis are interestingly complex. For example, Jevons, among others, believed that the social dimension of economic analysis is the subject matter of the science of the evolution of social relations, i.e. the newly established science of sociology. According to Jevons (1871, p. 20), "… instead of converting our present science of economics into an historical science, utterly destroying it in the process, I would perfect and develop what we already possess and at the same time erect a new branch of social science on an

historical foundation. This new branch of science ... is doubtless a portion of what Herbert Spencer calls Sociology, the Science of the Evolution of Social Relations."

The basic underlying assumptions employed by Jevons and Walras, i.e. an individual economic agent and the utility maximization principle, together with the structure of mathematical reasoning in which implicit results can be inferred from the assumed conditions, made the machinery of Marxian economics irrelevant in the newly established discipline of classical mathematical economics. This may explain the fact that despite having topics on value, Jevons (1871) and Walras (1874) could have managed their arguments on this subject without the necessity of making even one single reference to Marx or his work on value[11]. In fact, the classical mathematical economics provided a proper context in which the whole body of Marxian economics could have been safely left to one side.

The concrete fact regarding the impact of classical mathematical economics on the tradition of economic analysis is that it reduced the study of real economic life to an abstract mechanical science of economics. However, the real driving force behind the formation of this new discipline has neither been a theoretical motive nor an empirical necessity; otherwise the eminent classical economists would have taken the initiative to adopt a mathematical approach in their economic analysis. No single acknowledged classical economist has played any role in the formation of classical mathematical economics: Cournot was a mathematician; Jevons studied mathematics, logic and chemistry; Walras studied mathematics and engineering; and Pareto was an engineer.

A question of theoretical significance which demands further attention is that one may postulate that the contributions of Jevons and Walras in the formation of classical mathematical economics may have been primarily a response to the multi-dimensionality approach of classical economists like Smith, Say or Mill. This argument can further be supported by the fact that the basic underlying assumption in classical mathematical economics, i.e. the behavior of an individual economic agent in utility maximization, was developed before the emergence of Marxian economics. In what follows, we provide evidences which do not substantiate this argument.

Recall that the prime objective of Jevons and Walras in initiating the new discipline of pure (scientific or mathematical) economics was to

provide a remedy to the prevailing chaotic state of economic studies. They have never mentioned directly that their ultimate objective in mathematical treatment of economics was to tackle socialism in general or the Marxian economics in particular. However, their writings imply that socialism had a remarkable position in the list of schools and doctrines responsible for this chaotic state. Let us briefly refer again to their writings on this point:

According to Jevons (1876) "One hundred years after the first publication of the *Wealth of Nation,* we find the state of the science to be almost chaotic. There is certainly less agreement now about what political economy is than there was thirty or fifty years ago." It is evident that Jevons did not mean that the prevailing chaotic state was the result of the contributions of Smith one hundred years before. Similarly, according to Walras (1874, English translation 1954, p. 471) "There are today heaven knows how many schools of political economy: the *deductive* school and the *historical* school, the school of *laisser-faire* and the school of *State intervention* or *Socialism of the Chair,* the *Socialist school* properly so-called, the *Catholic* school, the *Protestant* school, etc."

We admit, however, that by adopting the principle of utility maximization of an individual economic agent as the basis of their mathematical approach, Jevons and Walras employed the instruments which were developed before the Marxian economics to establish the discipline of pure or scientific economics which completely made the machinery of Marxian economics futile and at the same time seriously weakened the explanatory power of classical economists by removing the class dimension from their analytical structures.

Nevertheless, Jevons and Walras used all the possible means of convincing classical economists that the new discipline of mathematical economics was highly productive. For example, in his introductory lecture at the opening session of 1876-1877 at University College, London, Jevons warned British economists strongly in the following words: "It may be safely asserted, however, that if English economists persist in rejecting the mathematical view of their science, they will fall behind their European contemporaries. How many English students, or even Professors, I should like to know, have sought out the papers of the late Dr. Whewell, printed in the *Cambridge Philosophical Transactions,* in which he gives his view of the mode of applying mathematics to our science? What English publisher, I may ask again, would for a moment

entertain the idea of reprinting a series of mathematical work on political economy? Yet this is what is being done in Italy by Professor Gerolamo Baccardo, the very learned and distinguished editor of the *Nuova Enciclopedia Italiano*... Now, too, that attention is at last being given to the mathematical character of the science, it is becoming apparent that a series of writers in France, Germany, Italy and England have made attempts towards a mathematical theory. Their work have been almost unnoticed, or, at any rate, forgotten, mainly on account of the prejudice against the line of inquiry they adopted ... On the present occasion, I cannot do more than mention the names of some of the principal writers referred to, such as Lang, Kroeneke, Buquoy, Dupuit, von Thunen, Cazaux, Cournot and Francesco Fuoco, on the continent; and Whewell, Tozer, Lardner, Peronnet Thompson, Fleeming Jenkin, Alfred Marshall and probably others, in Great Britain", Jevons (1876, pp. 199-200)

The efforts and contributions of Jevons and Walras did not, after all, convince the community of classical economists to apply mathematical methods in their economic analysis. However, theoretical development in the classical mathematical economics since then clearly signifies that this new discipline had the potential of isolating the Marxian economics from the mainstream theoretical work in economics.

6. Conclusion

Advances in classical and Marxian economics in the 19th century had produced a state of multi-dimensionality (or the so-called chaotic state) in economic analysis in which economic issues were studied in relation to historical, political and social sub-systems. Classical mathematical economics emerged following an attempt by a number of mathematicians and engineers to establish a pure mechanical economic science known alternatively as scientific or mathematical economics.

We have critically examined in this paper the nature of classical mathematical economics as well as its potential as a remedy to multi-dimensional political economy. Our argument is carried out with direct references to the work of Cournot, Jevons and Walras.

I have proposed for the first time the idea that classical mathematical economics was a response to Marxian economics. By direct references to Jevons and Walras, I have provided evidences to support this claim. I have shown that the sub-divisionism, so strongly advocated by Jevons and Walras as a remedy to the so-called "chaotic state of multi-

dimensionality" in economics, was successful enough in making Marxian machinery futile or essentially irrelevant within the context of pure or mathematical economics. By abstracting economic dimension from the real-life economic performance and ignoring the feedback mechanism and structural dependencies of economic dimension with political, historical and social dimensions, classical mathematical economics necessarily depleted the real content of fundamental economic concepts and thus reduced them to abstract notions suitable for mathematical manipulations.

The erroneous logic of abstraction in classical mathematical economics made it possible to leave the whole body of Marxian economics to one side. This is exactly the consequences of what Jevons and Walras and most of their disciples have effectively done in their economic contributions: despite having long discussions on issues like "value", they were successful in managing their arguments without even a single reference to Marx and his work on this topic.

Classical mathematical economics and particularly the contributions of Cournot, Jevons and Walras did not convince the community of classical economists to apply mathematical methods in their economic analysis. This is mainly due to the serious shortcomings in the methodology of abstracting pure economic behavior of representative individuals amenable to mathematical manipulations. This explains why there were not any significant advances in classical mathematical economics after Jevons and Walras. The revival of classical mathematical economics within the new discipline of modern mathematical economics emerged in the 1930's by a substantial methodological improvement, i.e. the integration of statistical techniques into mathematical economic analysis, whose analysis is beyond the scope of this paper.

And finally, my approach in this paper has been somewhat novel in the sense that I have only relied upon the writings of Jevons, Walras and to some extent Cournot in shaping the argument within a historical context to arrive at my conclusions, which to the best of my knowledge no author before me has derived them. Hence, these conclusions are quite controversial and disputable, demanding further research on this topic.

Endnotes

1- Civa (1711) is generally agreed to be the first true application of mathematical symbols, definitions and methods in economic analysis.

2- For a list of 38 works before Cournot, i.e. during the period 1711-1838 published on mathematical economics see Jevons's List of Mathematico-Economic Books, Memoirs and Other Published Writings, pp. 322-339 in his *Theory of Political Economy*, 1871.

3- We have mentioned here Marx's main contributions which were published before Jevons (1871) and Walras (1874).

4- A11 references to Cournot (1838) made in this paper are from its English translation by Nathaniel T. Bacon: *Researches into the Mathematical Principles of the Theory of Wealth*, New York: Macmillan, 1897, reprinted 1927.

5- See page *xvi*, preface to the 2nd edition of his book published originally in 1871.

6- Reprinted in his book: *The Principles of Economics*, 1905, pp. 198-199.

7- Walras has referred to Hermann Heinrich Gossen who published his book *Entwickelung der Gesetze des Menschlichen Verkehrs, und der daraus fliessnden Regeln fur menschliches.* (*The Laws of Human Relations and the Rules of Human Actions Derived Therefrom*) in 1854. It is interesting to note that "[Gossen] remained an obscure civil servant all his life. His book, of which there is still a copy in the British Museum -the only one in existence possibly- was accidentally discovered by Professor Adamson, and Stanley Jevons was again the first to recognize its merits.", see Gide and Rist (1909, 1948), p. 491.

8- Like Walras, Charles Roos, a founder of the *Econometric Society* in 1930, (with Ragnar Frisch and Irving Fisher), has confused the structural shortcomings of mathematical economics with the number of explanatory variables in a behavioral equation. In an article published in *Econometrica* 2(1), 73-74, he stated that "So many mathematical economists -Cournot, Walras, Pareto, Fisher, Frisch, Evans, Schultz and others- have already given such excellent reasons for employing mathematics in economics that it seems unnecessary for me to add anything. However ... some economists and others have said that there are so many variables involved in a study of human behavior that it will

never be possible to develop a science of economics. To these who would use these as an argument for not using mathematics in economics, one might reply that because there are so many variables, there is all the more need for an exact language to keep track of them."

9- The unsatisfactory arguments put forward by Cournot, Walras, Jevons and von Neumann and Morgenstern in explaining the origin of classical mathematical economics may further support this claim. (see Derakhshan, 2014)

10- It should be mentioned that Marx's work entitled *Grundrisse der Kritik der Politischen Oekonomie* or *Foundations of a Critique of Political Economy* known as *Grundrisse* was written during 1858-59 or 1857-1861 (i.e. before Jevons, 1871) but was unknown until its publication for the first time in the former Soviet Union in 1939. "Grundrisse" is a German word which means the "foundations or outlines". Moreover, volume 2 of *Capital* entitled *The Process of Circulation of Capital* and volume 3 of *Capital* entitled *The Process of Capitalist Production as a Whole* were edited by Friedrich Engels after Marx's death in 1883 and were published in 1885 and 1894, respectively. *The Theories of Surplus Value* in three volumes, known as volume 4 of *Capital* was written during 1861-63 and edited by Karl Kautsky 25 years after the death of Marx.

11- Jevons did not have any chapter on value in his *Theory of Political Economy* (1871). However, chapter *vii* in his *Principles of Economics* (1905) is on value. Chapter 16 (or Lesson 16) in Walras's *Elements of Pure Economics* is on the "Exposition and refutation of Smith's and Say's doctrines of the origin of value in exchange". Neither in this chapter, nor anywhere else in his book, has Walras made any reference to Marx or the Marxian theory of value.

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